Guthrie, Aaron 2009. Reformulating an Argument of Aristotle's against Contradictions.
In M. Rossetto, M. Tsianikas, G. Couvalis and M. Palaktsoglou (Eds.) "Greek Research in Australia: Proceedings of the Eighth Biennial International Conference of Greek Studies, Flinders University June 2009". Flinders University Department of Languages - Modern Greek: Adelaide, 27-35.

# Reformulating an Argument of Aristotle's against Contradictions

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Aristotle put forward a number of arguments against contradictions being true, in *Metaphysics*. However, many of them share a common flaw; the opponent in the debate (a dialetheist) can accept both the conclusion, *and* its negation. My aim will be to reformulate *one* argument, the Anscombe/Cresswell argument, to eliminate this flaw. I do so by exploiting modern developments in dialethic theory. I turn the argument into a non-question-begging *reductio* by exploiting the fact that a *reductio* can be to *absurdity* but not *contradiction*, and can conclude in the *rejection* of what lead to it (in this case, a contradiction). I also respond to a number of other objections to this argument, exploring the possibility that there is a good argument that keeps to the spirit of the original. I conclude that there is such an argument, but one that is only about very specific contradictions.

## 1. Introduction

In *Metaphysics* Aristotle put forward various arguments which conclude that all (or some) contradictions are not true. Aristotle's opponents in this debate did not have as developed a position as their modern counterparts. Moreover, the tools for engaging in meaningful debate, given the particular subject matter, had not been developed. A major aim of this paper is to look at one argument given by Aristotle in light of modern developments, and in doing so giving that argument a better chance of succeeding in showing something. As Priest (2006a:9–20) has pressed, many of the arguments that Aristotle gives against the view that there are true contradictions (i.e. dialetheism) share a flaw; the conclusion can be accepted by a dialetheist if it is in the form $\neg(A \& \neg A)$. This is so because a dialetheist can accept both $(A \& \neg A)$ and $\neg(A \& \neg A)$; a contradiction, but contradictions can be accepted by a dialetheist (see e.g. Priest, 2006a:19–20; Restall, 2004:81; Brady, 2004:41–42). This flaw seems to show that such arguments have uninteresting conclusions since the conclusion can be accepted by the *opponent* in the debate. But, I contend, in at least one argument the flaw is easily remedied by incorporating modern developments.

The argument I will examine is the "Anscombe/Cresswell"[1] argument, which I will *reformulate* in a simple way to avoid this flaw. I do the work of setting up the modern theory in section 2. This will allow me to reformulate the argument into a *reductio ad absurdum* (hereafter simply *reductio*), where the absurdity is not an absurdity *qua* contradiction. This will enable the argument to be a non-question-begging *reductio*, which concludes with the *rejection* of the assumption that lead to the absurdity; in this case the assumption will be a contradiction. This results in an argument without the above flaw, as a dialetheist cannot, under normal circumstances, accept and reject one and the same statement (Priest, 2006b:96–99).

In section 3 I reformulate the Anscombe/Cresswell argument. But beyond simply reformulating it to deal with the above flaw I also explore the possibility that there is a *good* argument here (or hereabouts). My method is to further reformulate the argument to deal with a number of objections put by Priest (2006a:26). As a first step, I note that aside from a *reductio* the argument can be considered a *conditional proof*, thus just showing *something* about *some* contradictions. I then go on to show how to reformulate the argument so as to not rely on the identity theory of predication. This move allows me to generalise the argument, which also allows me to deal with the objection that the argument relies on essentialism. I conclude that there is a good argument here, but that the argument concerns a specific sort of contradiction.

## 2. Situating the debate within the modern context

The broad context of the debate concerns the law of non-contradiction (LNC). Dialetheists reject LNC, while non-dialetheists accept it. But if the law is construed as accepting $\neg(A \,\&\, \neg A)$ for all A, then there is a problem; the dialetheist *can also accept this* (see e.g. Priest, 2006a:19–20; Restall, 2004:81; Brady, 2004:41–42). This commits them to a contradiction, but contradictions are not, *qua* contradiction, in conflict with a dialetheist position. This is not to say that a dialetheist should accept *all* contradictions. However, Priest (2006b:74–76) is one dialetheist who contends that this *particular* contradiction is true. This sort of dialetheist position is the one I will engage with, as it is in the best position to push the main objection of interest to this paper.

The objection, put against a number of Aristotle's arguments, is that the conclusion can be accepted by the opponent in the debate (Priest, 2006a:20ff.). This is so since a number of Aristotle's arguments conclude with a statement of the form $\neg(A \,\&\, \neg A)$, or a statement that is similarly problematic (e.g. $\neg\Diamond(A \,\&\, \neg A)$). But, as with the formulation of LNC above, a dialetheist can accept this and its negation. A way to deal with the problem in the LNC case is to formulate the law as the *rejection* of (A

[1] This interpretation of the argument can be found in Anscombe and Geach, 1967, in a chapter credited to Anscombe, and was developed and formalised in Cresswell, 2003.

& ¬A) for all A, rather than the *acceptance* of ¬(A & ¬A) for all A (Restall, 2004:81; Brady, 2004:44–47). Similarly, I exploit this distinction in responding to this objection.

## 2.1. Rejection and negation

Accept A, and you should reject ¬A, accept ¬A, and you should reject A, or so some might argue (Grim, 2004:61–63; Weir, 2004:403). Priest (2006b:96–99) contends this account of rejection is wrong; one can rationally accept a statement *and* its negation. Priest (*loc. cit.*) further holds that if one accepts a statement, one *need not* reject the statement's negation. Crucially, Priest (*loc. cit.*) contends that a dialetheist may employ rejection in the normal way, i.e. a dialetheist should not, at least under normal circumstances (but see Priest 2006a:109–110), both accept and reject one and the statement. This account has the virtue of allowing a polemic formulation of the law of non-contradiction, *viz.* as the *rejection* of (A & ¬A) for all A. Similarly for the argument reconstruction that I do, the argument will change from one that concludes with the negation of a contradiction, to a *reductio* that ends in the *rejection* of a contradiction.

## 2.2. Reductio arguments in this context

*Reductio* arguments, given that we should not be assuming that we should reject all contradictions, might be thought to beg the question (see e.g. Lukasiewicz, 1979:57). *Reductio* arguments can work like this: assume A, from A deduce a *contradiction*, conclude ¬A. If the non-dialetheist is right, then contradictions are never rationally acceptable, so deducing one means that you have deduced an absurdity (Priest, 2006a:86). By "absurdity" here I simply mean something not rationally acceptable. Moreover, that A entails an absurdity means you should *reject* A (Priest, 2006a:86). But in this context it cannot be assumed that contradictions are, *ipso facto*, absurdities, on pain of begging the question.

Priest (2006a:14), in fact, contends that some absurdities are worse than some contradictions; an example Priest (*loc. cit.*) gives is the belief "that I am a frog". The non-dialetheist need not agree with Priest on this point. All the non-dialetheist needs to do is contend that some statements are not rationally acceptable for reasons other than being (or entailing) contradictions. For example, if from a *reductio* they deduce that 1 = 2, they need to point to *that* as being an absurdity, even if they think that the associated contradiction is *worse*.[2] At this point one can give reasons that the purported absurdity is in fact an absurdity, as long as the reason is not the truth of the negation of the absurdity. Using the above example, one may also argue that if 1 = 2,

[2] This is not to say that all non-dialetheists need think that if A is abusrd, ¬A is true. Some may think that both A, ¬A are absurd in some cases (for instance, where A is the liar sentence).

all real numbers would be identical[3] and that this is absurd. One may further argue that this would make mathematics useless, and claim that this is absurd. One may also neglect to give reasons, and simply contend that 1 = 2 is self-evidently absurd.[4]

Given this, it is possible to run a non-question-begging *reductio* as long as the absurdity is not an absurdity *qua contradiction*. If, from the assumption of A an absurdity follows, then one still ought *reject* A (Priest, 2006b:104). An example of this can be found in the literature on Aristotle's arguments. Priest (2006a:24) interprets one of Aristotle's arguments as such a *reductio*:

> (i) if "man" refers to the substance of man, it refers to one thing; (ii) hence if "man" and "not man" meant the same thing "man" would refer to the substance of "not man"; (iii) but the substances (meanings) of "man" and "not man" are different...hence "man" would refer to (at least) two things.

But then one thing could be two things, and this could reasonably be claimed an absurdity, and on those grounds rejected along with what lead to it, namely that "man" and "not man" mean the same (Priest, 2006a:24–25). Below I will note how the Anscombe/Cresswell argument seems to have a similar first premise to this one, in so far as it relies on quantity.

## 3. The Anscombe/Cresswell argument

For the Anscombe/Cresswell argument to be understandable, something needs to be said about the technical terms that appear in it, namely "signification", and "substance" (Dancy, 1975:106–108). Aristotle's notion of substance is subtle, complicated, and controversial (see e.g. Cohen, 2009). However, for my analysis of the argument it will suffice to give a simple gloss of Aristotle's account of substance. Substances are the basic particulars of the world. They are to be identified with essences. They are things like man, tree, horse, and are properties that a thing has necessarily. They are to be distinguished from accidental properties, things like tall, bearded and so on, which are properties a thing has contingently.

With regards to signification it seems to me that the easiest way to explain it is via the distinction between *types* of signification. The two types being "signification of one" and "signification about" (Cresswell, 2003:181), which correspond to *a what* of predicating, and *a way* of predicating. The *what* in the former case corresponds to predicating substances, and the latter to predicating accidents (Kirwan, 1971:96–97). For the purposes of this paper the relevant notion is the "signification of one" type. In spelling out the *way* it predicates, I follow Priest's (2006a:25–26) and Kirwan's (1971:96–100) account, as it is a simple way to approach the text, and Priest is the

[3] See Mortensen (2000:205) for Dunn's argument to this effect.

[4] It may also be rational to, in some cases, move from A being self evidently absurd, and thus rejecting it, to accepting ¬A.

one with whom I am engaging. On this account what Aristotle means by "Socrates is a man" is that Socrates and man are the *same thing* (i.e. identical with each other) (Priest, 2006a:25–26; Kirwan, 1971:96–100). I will call this the "identity theory" of predication (Priest, 2006a:26).

With the preliminaries out of the way I can now introduce the Anscombe/Cresswell argument:

1. If F and G are both substance predicates, then if x is F, and x is G, F and G signify the same thing (they both signify x)
2. F and ¬F never signify the same thing
3. So, no x is both F and ¬F (1,2 by *modus tollens*)

(Cresswell, 2003:168; Priest, 2006a:25)

Premise 1 is justified by the identity theory of predication. If F and G are predicated as substances of x, they are identical to x. And if x is the same thing as F, and x is the same thing as G, then F is the same thing as G, by substitution of identicals. Priest (2006a:25–26) makes a few objections to this argument. I leave aside objections to the argument as a good *interpretation* of Aristotle as I am interested in the argument on its own grounds. I respond in turn to the following objections: it is only an argument against some contradictions; the conclusion can be accepted by a dialetheist; the argument relies on a bad theory of predication; the argument relies on essentialism; F and ¬F are never both substance predicates according to Aristotle. At the outset I concede that this argument only deals with some contradictions, not all, and as such will not refute dialetheism. However, as I argue below, it can still show something about some contradictions.

## 3.1. Reductio version of Anscombe/Cresswell

As Priest (2006a:26) points out, the conclusion seems to be ineffective, at least against some dialetheists; they can accept the conclusion of the Anscombe/Cresswell argument, *and* accept its negation. I now implement my promise of turning the argument into a non-question-begging *reductio*:

1. If F and G are both substance predicates, then if x is F, and x is G, F and G signify the same thing
2. ¬(No x is both F & ¬F) (assume for RAA)
3. Some x is both F & ¬F (2)
4. F and ¬F can signify the same thing (1, 3 by *modus ponens*)
5. But 4 is an absurdity, so there is good reason to *reject* the statement that lead to it, namely ¬(No x is both F & ¬F)

In the original form of the argument, accepting the conclusion, ¬(No x is both F & ¬F), did not rule out accepting the ¬(No x is both F & ¬F). However in this form the conclusion is the *rejection* of ¬(No x is both F & ¬F), and this *does* rule out accepting that contradiction. And this *reductio* is not question-begging as there is no claim that 4 is an absurdity *because* it entails a contradiction. Here the claim is *just* that 4 is an absurdity; it is an absurdity on its own merits.

There are a number of possible moves here, such as, deny that 4 is an absurdity, or reject 1. My aim is not to delve into deeper questions about the argument, such as whether 4 is an absurdity. The important point for this paper is that the dialetheist doesn't get to accept 4 *for free*, in the way they did in the original form of the argument. So putting this as an argument against (some) contradictions is not question-begging. Moreover, even if such moves worked, the argument would still show *something* about some contradiction, *viz*. that 4 follows from said contradiction. So regardless of what we want to say about 4, the argument is still *interesting*; it is at least an argument that if a certain contradiction obtained, then something particular to that contradiction would follow. That is to say, one may consider it as a *conditional proof* argument, i.e. from the assumption of 3, 4 follows, so (3 → 4). In this case, from the assumption of 2, 4 follows, so (2 → 4).

This method of turning an argument which is susceptible to Priest's objection into a *reductio* which isn't is quite simple, and so it *might* work for other arguments that face the same objection. The method is to simply take the premises of one of Aristotle's arguments, assume the negation of the conclusion (i.e. a contradiction), and then see if one can deduce an absurdity.

## 3.2. Identity theory of predication

Priest (2006a:25–26) also objects that the argument relies on the identity theory of predication. However, it is possible to modify the argument so as to avoid commitment to the identity theory of predication, whilst keeping to the spirit of the original argument:

1d) $\forall F \forall x[Fx \rightarrow \forall G(Gx \rightarrow F=G)]$[5] (given that F and G are substance predicates) (c.f. Russell, 1905:490)

What this says is that, for all F and all x, if any x is an F, and a G, then F and G are the same, which seems to be roughly the same as the original. But there is no commitment here to the identity theory of predication. On that theory a predicate came

[5] Other formalisations seem possible, for example, following Noonan's (1977:163) account of signification, one might plug in $\Box\forall x(Fx\equiv Gx)$ in place of (F=G). I don't pursue this here, as Noonan (1977:165) constructs this so as to allow substitution *salva veritate*, and this complicates things in the current context; as for any contradiction, it seems that A is substitutable *salva vitate* for ¬A (since both are true and false).

out as being identical with a subject (i.e. F being identical with x), but this argument is not committed to that claim.

At this point one might ask what *justifies* the first premise if it makes no appeal to the identity theory of predication.[6] My response here is to point out that it applies to all objects that fall under a certain description, *viz*. things that take only one substance predicate. Moreover, replace "substance predicate" by some other specification of a set of predicates, and then the new argument will apply to objects which take only one of those predicates. The argument could be about, for instance, things that have only one colour. Note also that on this construal, the argument is similar to the argument about meanings given above. This is because the focus of the argument is on things that are only one thing *qua* a given set of predicates, and the meaning argument was about words which have only one meaning. This also dismantles the commitment to essentialism. However, the *importance* of the argument is a metaphysical and ontological issue in the following sense; *what* things are only one thing (*qua* a given set of predicates) is a metaphysical and ontological issue. This is where an argument that there are things, such as substances, that can only be one thing *qua* a certain sort of predicate would make the argument *metaphysically* and *ontologically* more interesting. And an argument that there are some metaphysically or ontologically interesting predicates would make the argument even more interesting.

Returning to the formulation as a *reductio*, one can get the following argument (I assume here M,¬M are substance predicates, I leave discussion of this assumption to the next section of the paper):

1d) $\forall F \forall x[Fx \rightarrow \forall G(Gx \rightarrow F=G)]$ (given that F,G are substance predicates)

2d) $\forall F \neg(F=\neg F)$ (a reformulation of premise 2 of Anscombe/Cresswell)

3d) $(Mx \ \& \ \neg Mx)$ (assume for RAA)

4d) $(M=\neg M)$ (1,3 *modus ponens* (with M, ¬M substituted for F and G respectively in 1)) (absurdity)[7]

Further, we can then derive:

5d) $\neg(M=\neg M)$ (2 by instantiation)

6d) $\neg(M=M)$ (4,5 by substitution) (c.f. Irwan, 1991:183)

6 I owe this point to Stephan Kubicki.

7 I am assuming here that I can substitute in ¬M for either G, or F. This seems reasonable, given that the argument goes by way of saying, take one *of these things* (whether you want to call them predicates, or whatever), such as M,¬M and so on (and I formalize the variable form of those things as F, G), only one of them applies to x.

## 3.3. Substance predicates

Priest (2006a:26) also objects that, for any F, F and ¬F cannot both be substance predicates on Aristotle's account, and, Priest claims, both are needed to be for the argument to go through. The first point to make is that this is a criticism of the argument with regards to it showing anything about *substances*. But it could still work for some predicates if, instead of specifying Aristotelian substances, some other predicates were specified which included for some F both F and ¬F, and all objects took only one of the specified predicates. So this again seems to be an issue with the metaphysical and ontological *importance* of the argument, which is not to say it is an unimportant issue. The second point to make is that instead of assuming just (Fx & ¬Fx), one can also assume that F and ¬F are both substance predicates, which will then be the assumption of a more complicated contradiction, but a contradiction nonetheless. Thus what will end up being rejected will still be a contradiction. What we have ended up with, then, is an argument that can at least show that if all objects take only one of a given set of predicates, and those predicates include some which are such that both they and their negation are in the set, then for those predicates which include both they and their negation, if any object takes both predicates, identity fails. While this is an argument about a very specific sort of contradiction, it is a good argument which doesn't suffer from any of the flaws claimed by Priest of the original, yet still keeps to the spirit of the original.

## 4. Conclusion

The major aim of this paper has been to show that it is possible to turn the Anscombe/Cresswell argument from one with a problematic conclusion, to one with a non-problematic conclusion. I did so by turning it into a non-question-begging *reductio*, however I also noted that one could reconstruct the argument into an interesting *conditional proof*. I also explored the possibility that there was a good argument that didn't stray too far from the Anscombe/Cresswell argument. I did so by reformulating the argument so as to deal with a number of criticisms made by Priest. I concluded that there is a good argument, which keeps to the spirit of the original argument, to be found here.

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